Scale effects

- Notice that in Romer (1990) and elsewhere,

$$g \sim \hat{A} = \gamma L_A^{\lambda}, \quad \text{where } L_A - \text{R\&D employment.}$$

- These are STRONG scale effects:
    - provided that the share of R&D employment is the same, bigger countries grow faster:

$$g \sim \gamma L^{\lambda} \cdot \underbrace{(\ell_A^{\lambda})}_{\text{fixed}}, \quad \ln g \sim \lambda \ln L.$$

    - clearly inconsistent with empirical evidence!
    - MOREOVER, if there is constant population growth, $\hat{L} = n$, then

$$g \sim \gamma (L_0 e^{nt})^{\lambda} \underbrace{(\ell_A^{\lambda})}_{\text{fixed}}.$$

↳ THE GROWTH RATE IS GROWING (at a rate λn).

- Jones (1995) has shown that also along the US time series, the evidence is inconsistent with strong scale effects:

  the R&D employment / expenditure increased greatly whereas the long-run growth rate has remained virtually unchanged.

  ↳ "JONES CRITIQUE".

Responses to the Jones critique

- Jones (1995) himself + followers (e.g., Kortum, Segerstrom) pose:

$$\dot{A} = \gamma L_A^{\lambda} A^{\phi} \quad , \quad \phi<1$$

[$\phi=1$ implies strong scale effects,
$\phi\in(0,1)$ is "standing on shoulders",
$\phi<0$ is "fishing out".]

- In this case, the long-run growth rate is

$$g \sim \hat{A} = \gamma L_A^{\lambda} A^{\phi-1} .$$

- Assuming BGP ($\hat{A}\equiv\text{const} \Rightarrow \hat{\hat{A}}=0$), we have:

$$0 = \lambda \hat{L}_A + (\phi-1)\hat{A}$$

$$g\sim \quad \hat{A} = \frac{\lambda}{1-\phi}(\underset{\overset{\parallel}{0}}{\hat{\ell}_A} + \hat{L}) = \frac{\lambda n}{1-\phi}$$

- The long-run growth rate is proportional to the population growth rate.

[• The class of models sharing this property is called SEMI-endogenous growth models.

- The long-run growth rate does not depend on any endogenous variable,
  - ↳ despite R&D in the model!
  - ↳ $n=0 \Rightarrow g=0$ $\boxed{!!!}$
  - ↳ Jones foresaw a major slowdown in the US economy, already around 2000 (e.g., Jones 2002, AER), perhaps we're observing it just now??

- "Second generation" R&D-based endogenous growth models (e.g., Young 1998, Aghion & Howitt 2000, Peretto 2000):

$$\dot{A} = \gamma \left(\frac{L_A}{L}\right)^{\lambda} A .$$

→ hence, $\underbrace{g \sim \hat{A} = \gamma \left(\frac{L_A}{L}\right)^{\lambda} = \gamma l_A^{\lambda}}$.

↳ LONG-RUN GROWTH RATE INDEPENDENT OF POPULATION SIZE & GROWTH

- Jones (1995) model has "weak scale effects" (level effects)
- 2nd generation endogenous growth models recover the endogeneity of the growth rate (l_A is a choice variable)

- These models are also called "non-scale" models.

Knife-edge conditions in growth models

- Take $\dot{A} = \gamma \frac{L_A^{\lambda}}{L^{\beta}} A^{\phi}$, where L^{β} is "product proliferation effect" (explained in the increasing variety framework)

- Endogenous growth requires
  $\phi = 1$ [with SCALE EFFECTS]
  or $\phi = 1$ and $\beta = 1$ [without SCALE EFFECTS]

- Jones (1999) criticises models based on knife-edge assumptions as implausible.

- The "linearity critique" (Jones 2003, 2005)

↳ any endogenous growth model has to contain an equation of form

$$\dot{X} = \underbrace{\alpha}_{\downarrow} X^{\phi}, \quad \text{where } X - ??, \quad \boxed{\phi = 1}$$

can be endogenous, typically is.

↳ $\phi \neq 1$, i.e, any deviation from pure linearity, is then leading to qualitatively different model dynamics

- Not entirely true! Take two state variables:

$$\begin{cases} \hat{x} = x^{\alpha} y^{\beta} \\ \hat{y} = x^{\gamma} y^{\delta}. \end{cases}$$

Assuming the BGP, $\hat{x} \equiv \text{const}$ and $\hat{y} \equiv \text{const}$ $(\hat{\hat{x}} = \hat{\hat{y}} = 0)$:

$$\begin{cases} 0 = \alpha \hat{x} + \beta \hat{y} \\ 0 = \gamma \hat{x} + \delta \hat{y} \end{cases} \Leftrightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \hat{x} = \hat{y} = 0 \quad \text{or} \quad \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \text{ is singular } (\alpha\delta = \beta\gamma).$$

- Better call this "SINGULARITY CRITIQUE".
- Singularity of a given matrix is a knife-edge condition.

- General argument (Growiec, 2007)

→ For any growth model of form $\dot{X} = F(X)$, $X(t)$ - vector, existence of a BGP requires making a knife-edge assumption.

↳ note that this case includes also higher-order differential equations

↳ $X(t)$ contains state variables only (reduced-form model).

Empirical evidence

US DATA 1950 -2000

OECD PANEL DATA

- Ha and Howitt (2007), Madsen (2008) find that the non-scale endogenous growth model is better aligned with data than the semi-endogenous growth model.

- Caveats:

↳ INTERNATIONAL TECHNOLOGY DIFFUSION

↳ TECHNOLOGY ADOPTION LAGS

↳ MULTI-DIMENSIONAL TC $\boxed{?}$
